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COMMENT

Comment on 'Garden of Eden states in a traffic model revisited'

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Abstract
Recently, Huang and Lin suggested a combination of two successful mean-field theories, the 2-cluster approximation and paradisical mean-field, for the Nagel–Schreckenberg cellular automaton model of traffic flow. We argue that this new approximation is inconsistent since it violates the Kolmogorov conditions.

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In a recent work [1] Huang and Lin have studied a combination of two mean-field theories, the 2-cluster approximation [2–4] and paradisical mean-field (PMF) [4, 5], for the Nagel-Schreckenberg cellular automaton model of traffic flow [6] (for a review, see [7]). The suggested combined theory yields results for the flow-density relation which are worse compared to Monte Carlo simulations than those of each individual approximation alone. The authors concluded that the success of paradisical mean-field theory is accidental and cannot be improved systematically, in contrast to the cluster approximation. In this Comment we will show that the combined theory as suggested by Huang and Lin is inconsistent since it violates the elementary Kolmogorov conditions. This also invalidates the conclusions about the success of paradisical mean-field theory.

In the 2-cluster approximation the probability $P(\tau_1, \ldots, \tau_L)$ of finding the system in the state $(\tau_1, \ldots, \tau_L)$ is factorized into 2-site probabilities $P_{\tau,\tau'}$:

$$P(\tau_1, \ldots, \tau_L) \propto P_{\tau_1,\tau_2} P_{\tau_2,\tau_3} \cdots P_{\tau_{L-1},\tau_L} P_{\tau_L,\tau_1}. \tag{1}$$

Here we assume translational invariance so that the probabilities $P_{\tau,\tau'}$ do not depend on the position.

An important consequence are the so-called Kolmogorov consistency conditions [8] which for the 2-cluster approximation read

$$\sum_{\tau_2} P_{\tau_1,\tau_2} = P_{\tau_1} = \sum_{\tau_2} P_{\tau_2,\tau_1} \tag{2}$$

where P_τ is the probability of finding a cell in state τ. Especially for $\tau = x$, denoting an empty cell in the notation of [1], we have $P_x = 1 - \rho$ where ρ is the total density of cars. Since also $P_1 + P_2 = \rho$, instead of equation (2) in [1] one has more precisely

$$\rho = P_{1x} + P_{11} + P_{12} + P_{2x} + P_{21} + P_{22} \tag{3}$$

$$= P_{x1} + P_{11} + P_{21} + P_{x2} + P_{12} + P_{22} \tag{4}$$

and

$$1 - \rho = P_{xx} + P_{x1} + P_{x2} \tag{5}$$

$$= P_{xx} + P_{1x} + P_{2x}. \tag{6}$$

We have analysed numerically the equations given in the appendix of [1] and found that these consistency conditions are violated. In fact, we found that the results depend strongly on the iteration scheme used. If one adjusts the normalization $\mathcal{N}$ during the iteration to ensure the normalization of probabilities (equation (1) of [1]) then the numerical solution converges to $\rho = 0$, even for initial values with $\rho \neq 0$. On the other hand, if one uses equation (1) of [1] to eliminate one of the equations in the appendix, the normalization condition is always satisfied during iteration. One then can use $\mathcal{N}$ to fix the density ρ according to equation (2) of [1]. In this case we find that the Kolmogorov consistency conditions (3)–(6) are violated and in general $\sum_\tau (P_{1\tau} + P_{2\tau}) \neq \rho \neq \sum_\tau (P_{\tau 1} + P_{\tau 2})$. It even seems that no solution exists, at least in certain density regimes.

In order to exclude the possibility that these problems are related to a mistake or typo in the set of equations given in [1] we have done an independent calculation. However, we find it much more convenient not to use the update ordering R2–R3–R4–R1 that was used in [2, 3] and adopted in [1]. Instead the original update order R1–R2–R3–R4 [9] is preferable since it allows a much easier identification of the GoE states and avoids the introduction of the weighting $\mathcal{W}$. It also turns out that this approach is numerically much more stable since no terms proportional to $\mathcal{N}^2$ (where $\mathcal{N}$ is the normalization) appear. For our system of equations we find the same type of behaviour as described above for the equations of [1].

The reason for this failure is that the modified equations after elimination of the GoE states do not automatically guarantee the conservation of cars, in contrast to the original 2-cluster equations. It is not possible to satisfy the normalization condition $\sum_{\tau_1,\tau_2} P_{\tau_1,\tau_2} = 1$ and the Kolmogorov conditions (2) by just introducing one normalization factor $\mathcal{N}$. One could try to use normalizitions $\mathcal{N}_{\tau,\tau'}$ depending on the state, but this would make the system of equations even more complicated than the 3-cluster approximation [2–4] which already treats most GoE states correctly.

Finally, we want to comment on the conclusions of [1]. We believe that the combined approach—if successful—must be considered as an improvement of the 2-cluster approach, not of PMF as suggested in [1]. If it were possible to carry out such an approximation consistently it would most certainly lead to an improvement of the 2-cluster results. In fact, the main point of the PMF is not the quantitative agreement with simulations but the fact that it allows us to determine the origin of correlations and thus to improve the understanding of the underlying physics. This was already emphasized in [4, 5].

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